Robotics

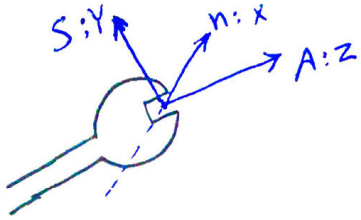
Section 4

Rotation Matrix

1- Orientation of Frame with another

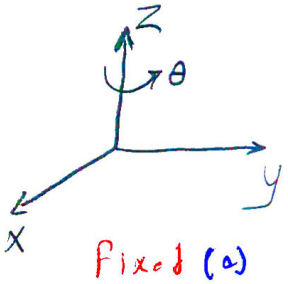
2- Coordinates of Point P with different Frame

3- Rotation Operator



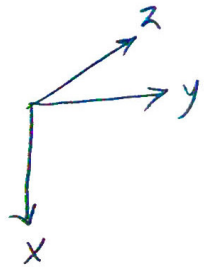
End Effector

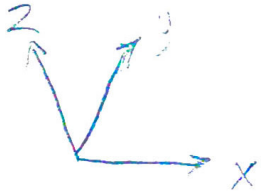
Composition and Rotaton



Fixed (a)

Moving (b)





around new y (C_1)

Post multiply

around fixed y (C_2)

Pre multiply

Based on

Successive rotation (similarty Transformation)

$$R_{C_1} = \underset{\text{Fixed}\atop\text{Pre multiply}}{R(z,\theta)} \; I \; \underset{\text{Current}\atop\text{Post multiply}}{R(y,\phi)}$$

$$R_{C_2} = R(y,\phi)\, R(z,\theta)\, I$$

Example

R: Specified by the following Sequence

1- rotation of θ about current X-axis

2- rotation of ϕ about current Z-axis

3- rotation of α about fixed Z-axis

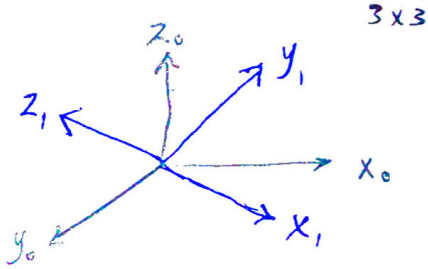
4- rotation of β about current Y-axis

5- rotation of δ about fixed X-axis

$$R = R(x,\delta)\, R(z,\alpha)\, I\, R(x,\theta)\, R(z,\phi)\, R(y,\beta)$$

Parametrization of Rotations

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}_{3\times 3}$$



الدوران في الفراغ بشكل عام يتم بتتبع متغيرات دورانات وتتم فيه تغيير مكان جميع المحاور

Euler Angles

(Z Y Z)

(Z B C)

(θ ϕ ψ)

Representation I

$$R = R(z,\theta)\, R(y,\phi)\, R(Z,\psi)$$

$$= \begin{bmatrix} C\phi\, C\theta\, C\psi - S\theta\, S\psi & \text{——} & C\phi\, S\theta \\ S\phi\, C\theta\, C\psi + C\theta\, S\psi & \text{——} & S\phi\, S\theta \\ -S\theta\, C\psi & S\theta\, S\psi & C\theta \end{bmatrix}$$

Euler Angles (ZXZ) (ZAC) representation II

$$R = R(z,\theta)\, R(x,\phi) R(z,\psi)$$

Roll - Pitch - Yaw
Z Y X

لتمثيل الدوران في الفراغ

*Fixed elementary axis

1- ψ around x
2- ϕ around y
3- θ around z

$$R = R(z,\theta)\, R(y,\phi)\, R(x,\psi)$$
